Math 308O, Midterm 1 Name: ______________________

Signature: ______________________________________

Student ID #: ______________ Section #: ______________

- You are allowed a Ti-30x IIS Calculator and one 8.5×11 inch paper with handwritten notes on both sides. Other calculators, electronic devices (e.g. cell phones, laptops, etc.), notes, and books are **not** allowed.
- *All* answers on the exam must be justified. You will receive at most 1 point out of 10 for an answer without any explanation.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Raise your hand if you have a question.

1	/10
2	/10
3a	/10
3b	/10
4	/10
5	/10
6	/10
T	/60

Good Luck!

(1) [10pts] Determine how many solutions the linear system has. If it has a solution find it and if the system has more than one solution, write down two different solutions.

$$\begin{aligned} x + 2y - z &= 1 \\ x + 2y \phantom{{}- z} &= 1 \end{aligned}$$

(2) [10pts] Determine a 2×3 matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ in *reduced echelon form*, such that z is a free variable and such that

$$\mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

is the general solution to the system

$$\begin{aligned} ax + by + cz &= -1 \\ dx + ey + fz &= 2. \end{aligned}$$

(3) In each of the following problems determine whether the set of vectors are linearly dependent or linearly independent. (*Note:* These questions should not require long and involved computations. If you find yourself doing a complicated computation, you may want to look for another approach.)

(a) [10pts]

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(b) [10pts]

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix} \right\}$$

(4) [10pts] Find an example of vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \in \mathbb{R}^2$ such that

$$\text{span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3) = \mathbb{R}^2 \quad \text{and} \quad \text{span}(\mathbf{u}_1, \mathbf{u}_2) \neq \mathbb{R}^2$$

OR explain why this is not possible. If you provide an example, your answer should include an explanation of why those vectors have the desired properties.

(5) Find an example of 4 linearly independendent vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4 \in \mathbb{R}^4$ such that the linear transformation

$$T\colon \mathbb{R}^4 \to \mathbb{R}^4,\ T(\mathbf{x}) = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + x_4\mathbf{a}_4$$

is *not* onto, OR explain why this is not possible. If you provide an example, your answer should include an explanation of why those vectors have the desired properties.